Inferential Statistics and Probability
a Holistic Approach

Chapter 5
Discrete Random Variables

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Random Variable

- The value of the variable depends on an experiment, observation or measurement.
- The result is not known in advance.
- For the purposes of this class, the variable will be numeric.

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Random Variables

- Discrete – Data that you Count
  - Defects on an assembly line
  - Reported Sick days
  - RM 7.0 earthquakes on San Andreas Fault
- Continuous – Data that you Measure
  - Temperature
  - Height
  - Time

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Discrete Random Variable

- List Sample Space
- Assign probabilities P(x) to each event x
- Use "relative frequencies"
- Must follow two rules
  - $P(x) \geq 0$
  - $\Sigma P(x) = 1$
- P(x) is called a Probability Distribution Function or pdf for short.

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Probability Distribution Example

- Students are asked 4 questions and the number of correct answers are determined.
- Assign probabilities to each event.

x	P(x)
0	.1
1	.1
2	.2
3	.4
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Probability Distribution Example

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1	.1
2	.2
3	.4
4	.2

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Mean and Variance of Discrete Random Variables

- Population mean μ, is the expected value of x

  $\mu = \Sigma[\ (x)\ P(x)\ ]$

- Population variance σ^2, is the expected value of $(x-\mu)^2$

  $\sigma^2 = \Sigma[\ (x-\mu)^2\ P(x)\ ]$

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Example of Mean and Variance

x	P(x)
0	0.1
1	0.1
2	0.2
3	0.4
4	0.2
Total	**1.0**

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Example of Mean and Variance

x	P(x)	xP(x)
0	0.1	0.0
1	0.1	0.1
2	0.2	0.4
3	0.4	1.2
4	0.2	0.8
Total	**1.0**	**2.5=μ**

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Example of Mean and Variance

x	P(x)	xP(x)	x-μ
0	0.1	0.0	-2.5
1	0.1	0.1	-1.5
2	0.2	0.4	-0.5
3	0.4	1.2	0.5
4	0.2	0.8	1.5
Total	**1.0**	**2.5=μ**	

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Example of Mean and Variance

x	P(x)	xP(x)	x-μ	$(x-\mu)^2$
0	0.1	0.0	-2.5	6.25
1	0.1	0.1	-1.5	2.25
2	0.2	0.4	-0.5	0.25
3	0.4	1.2	0.5	0.25
4	0.2	0.8	1.5	2.25
Total	**1.0**	**2.5=μ**		

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Example of Mean and Variance

x	P(x)	xP(x)	x-μ	$(x-\mu)^2$	$(x-\mu)^2P(x)$
0	0.1	0.0	-2.5	6.25	.625
1	0.1	0.1	-1.5	2.25	.225
2	0.2	0.4	-0.5	0.25	.050
3	0.4	1.2	0.5	0.25	.100
4	0.2	0.8	1.5	2.25	.450
Total	**1.0**	**2.5=μ**			**1.450=σ^2**

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Bernoulli Distribution

- Experiment is one trial
- 2 possible outcomes (Success,Failure)
- p=probability of success
- q=probability of failure
- X=number of successes (1 or 0)
- Also known as Indicator Variable

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Example - Bernoulli

- A basket player makes 70% of free throws. One shot is taken.
- p = P(success) = 0.70
- q = P(failure) = 1-p = 0.30

x	P(x)	xP(x)	$(x-\mu)$	$(x-\mu)^2$	$(x-\mu)^2P(x)$
0	0.30	0	-0.70	0.49	0.147
1	0.70	0.70	0.30	0.09	0.063
Total	**1.0**	**μ=0.70=p**			**σ^2=0.21=pq**

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Mean and Variance of Bernoulli

x	P(x)	xP(x)	$(x-\mu)^2P(x)$
0	(1-p)	0.0	$p^2(1-p)$
1	p	p	$p(1-p)^2$
Total	**1.0**	**p=μ**	**$p(1-p)=\sigma^2$**

- $\mu = p$
- $\sigma^2 = p(1-p) = pq$

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Binomial Distribution

- n identical trials
- Two possible outcomes (success/failure)
- Probability of success in a single trial is p
- Trials are mutually independent
- X is the number of successes

- Note: X is a sum of n independent identically distributed Bernoulli distributions

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Example - Binomial

- A basket player makes 70% of free throws. three shots are taken. Find the probability of making exactly 2 Shots

- p = P(Success) = 0.70
- q = P(Failure) = 1-p = 0.30
- n = number of independent trials =3

- P(SSF) = P(S)P(S)P(F) = (0.70)(0.70)(0.30) = 0.147
- P(SFS) = P(S)P(F)P(S) = (0.70)(0.30)(0.70) = 0.147
- P(FSS) = P(F)P(S)P(S) = (0.30)(0.70)(0.70) = 0.147

- P(X=2) = 3(0.147)= 0.441

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Binomial Distribution

- n independent Bernoulli trials
- Mean and Variance of Binomial Distribution is just sample size times mean and variance of Bernoulli Distribution

$$p(x) = {}_nC_x p^x (1-p)^{n-x}$$

$$\mu = E(X) = np$$

$$\sigma^2 = Var(X) = np(1-p)$$

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Binomial Examples

- The number of defective parts in a fixed sample.
- The number of adults in a sample who support the war in Iraq.
- The number of correct answers if you guess on a multiple choice test.

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Binomial Example

- 90% of super duplex globe valves manufactured are good (not defective). A sample of 10 is selected.
- Find the probability of exactly 8 good valves being chosen.
- Find the probability of 9 or more good valves being chosen.
- Find the probability of 8 or less good valves being chosen.

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Using Technology

X	p(X)	cumulative probability
0	0.00000	0.00000
1	0.00000	0.00000
2	0.00000	0.00000
3	0.00001	0.00001
4	0.00014	0.00015
5	0.00149	0.00163
6	0.01116	0.01280
7	0.05740	0.07019
8	0.19371	0.26390
9	0.38742	0.65132
10	0.34868	1.00000
	1.00000	

9.000 expected value
0.900 variance
0.949 standard deviation

Use Minitab or Excel to make a table of Binomial Probabilities.

$P(X=8) = .19371$

$P(X\leq 8) = .26390$

$P(X\geq 9) = 1 - P(X\leq 8) = .73610$

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Poisson Distribution

- Occurrences per time period (rate)
- Rate (μ) is constant
- No limit on occurrences over time period

$$P(x) = \frac{e^{-\mu}\mu^{x}}{x!} \qquad \mu = \mu \qquad \sigma = \sqrt{\mu}$$

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Examples of Poisson

- Text messages in the next hour
- Earthquakes on a fault
- Customers at a restaurant
- Flaws in sheet metal produced
- Lotto winners

Note: A binomial distribution with a large n and small p is approximately Poisson with $\mu \approx np$.

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Poisson Example

- Earthquakes of Richter magnitude 3 or greater occur on a certain fault at a rate of twice every year.
- Find the probability of at least one earthquake of RM 3 or greater in the next year.

$$\begin{aligned} P(X>0) &= 1-P(0) \\ &= 1-\frac{e^{-2}2^{0}}{0!} \\ &= 1-e^{-2} \approx .8647 \end{aligned}$$

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Poisson Example (cont)

- Earthquakes of Richter magnitude 3 or greater occur on a certain fault at a rate of twice every year.
- Find the probability of exactly 6 earthquakes of RM 3 or greater in the next **2** years.

$$\mu = 2(2) = 4$$

$$P(X = 6) = \frac{e^{-4}4^{6}}{6!} \approx .1042$$

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